MATH 4341 – Introduction to Complex Variables

Fall 2021

Instructor: Dr. Scott M. LaLonde
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Office hours: Tuesdays & Thursdays 1:00–2:30 P.M., or by appointment. All office hours will be conducted remotely via Zoom (link available on Canvas).

Scheduled lectures: MWF, 1:25–2:20 P.M. in RBN 3041.

Course Information

Course Webpage: All course information and documents will be posted on Canvas.

Textbook: *Fundamentals of Complex Analysis with Applications to Engineering and Science* (Third Edition) by Saff and Snider (ISBN: 978-0134689487).

Prerequisites: A grade of C or better in MATH 3404 and MATH 3425 (or equivalent).

Course Description: Study of geometry of the complex plane and functions of a single complex variable, with particular emphasis given to analytic functions, differentiation, and integration. Topics covered will include the Cauchy-Riemann equations, harmonic functions, Cauchy's theorem and the Cauchy integral formula, Taylor and Laurent series expansions, and residue calculus. Special topics may include conformal mappings and applications to partial differential equations.

Student Learning Outcomes: Upon completion of this course, students should be able to do the following:

- Formulate correct, rigorous mathematical proofs of results in the realm of complex analysis.
- Articulate relevant mathematical ideas correctly and precisely, both verbally and in writing.
- Understand the different ways of representing complex numbers (both arithmetically and geometrically) and how to work with functions of a complex variable.
- Use the Cauchy-Riemann equations to prove that a complex function is differentiable.
- Evaluate integrals of complex functions using Cauchy's theorem, the Cauchy integral formula, and the residue theorem.
- Use contour integrals to evaluate integrals of real-valued functions that are otherwise difficult to handle.
- Compare and contrast the concepts of continuity, differentiability, holomorphicity, and analyticity of functions of a complex variable.
- Compute the Taylor series of a holomorphic function and determine its disk of convergence.
- Compute the Laurent series of a meromorphic function and determine its annulus of convergence.
- Classify the singularities of a function of a complex variable, and understand the connection to the Taylor/Laurent series of the function.

Assignments and Grading

Homework

Homework will be assigned more or less on a weekly basis. Abstract mathematics is best learned through practice, so it is imperative that you make an honest effort to complete each assigned problem. Homework assignments need to be written legibly or typed, and all proofs must be written in complete sentences. I reserve the right to reject any papers that are illegible. Homework will generally be collected on Wednesdays via Canvas.

Each assignment will consist of both proofs and computational problems. I will usually divide assignments into three sections according to the difficulty of the problems:

- **Easy:** These problems are designed to check your understanding of the key concepts. They may involve routine calculations, very simple proofs, and generating and/or analyzing examples and counterexamples.
- **Medium:** The problems in this section are a little more involved. They may require you to perform more complicated calculations, or to prove new facts using the recent course material.
- **Hard:** These problems are fairly difficult. They likely require some creativity, or you may need to explore concepts beyond what we have done in class.

You have hopefully come to learn that an attempt at writing a mathematical proof either ends with a correct proof, or it does not. However, I will grade each proof problem on a scale of 0 to 4 to allow for partial credit. The score will depend on both content and presentation as follows:

- **4:** You have constructed a correct proof of the given statement, and it is written clearly, coherently, and in complete sentences.
- **3:** Your solution is mostly correct, but there are some small defects that keep your argument from being completely airtight. It is also written clearly and in complete sentences.
- **2:** You are headed in the right direction, but there are fundamental flaws in the argument or exposition. Your proof is partially correct, or you've cut corners in the written presentation.
- **1:** Your proof is fundamentally flawed and/or poorly written.
- **0:** Your proof is completely incorrect or incoherent, or you did not make a reasonable attempt at solving the problem.

Other problems (e.g. computational ones) will probably be graded on different scales, which will vary according to the complexity of the problem.

Exams

There will be two exams during the semester, which will be given during our usual class time. The tentative dates for these exams are:

- **Exam 1:** October 1
- **Exam 2:** November 12

Each exam will likely consist of a combination of conceptual questions (i.e. questions involving definitions, theorems, and examples), computational questions, and some short proofs. We will discuss the content and structure of each exam about one to two weeks prior to the scheduled date.

In addition, there will be a comprehensive final exam given at the end of the semester. The designated time for this exam is:

- Friday, December 10, 12:30–2:30 P.M. (Location TBD)

The format of the final exam will be similar to that of the two semester exams.

Grading

Your grade for this course will be computed as follows. The scale for determining your letter grade will be no more harsh than the traditional one shown below.

Assignment	Total %
Homework	25
Exam 1	25
Exam 2	25
Final Exam	25
Total	100

Numerical	Letter
90 – 100	A
80 – 89	B
70 – 79	C
60 – 69	D
Below 60	F

Course Policies

Canvas

You must activate your Canvas account and check it regularly. You can activate your account and log in at `https://www.uttyler.edu/canvas`. If you are registered for the course, then you should already have access to the Canvas page. All announcements and important documents will be posted there.

Email

Along with the built-in Canvas messaging system, the preferred means of communication for this course is official UT Tyler email. If you email me, it needs to be sent from your Patriots account to my UT Tyler email address (`slalonde@uttyler.edu`). In the event that I need to contact you, I will send an email to your Patriots account, and I will assume that you have read any such message.

Office Hours

I have regularly scheduled office hours, which are set aside as time for you to talk to me about the course. Attending office hours should be your first course of action if you find that you are struggling. You should not be afraid to come ask me questions when you are studying or working on homework. This course moves quickly—*don't let yourself fall behind*. If you are unable to attend my usual office hours, you can always set up an appointment or ask questions via email.

Attendance

Attendance is not officially required, and it is not factored into your overall grade. However, It will be very hard for you to succeed in this course if you do not attend class and keep up with the material—poor attendance will likely affect your grade indirectly by impacting your performance on the graded assignments.

Make-up Policy

Make-ups and extensions on assignments will only be granted in the case of severe illness, absences that are required as part of a UT Tyler obligation, or for religious observances. You need to notify me as soon as possible (or at least one week ahead of time in the case of a planned absence) and provide appropriate documentation. Other makeups and extensions are granted only in extreme cases and at the discretion of the instructor.

Collaboration, Plagiarism, and Academic Dishonesty

I encourage you to talk to your classmates when studying and working on homework assignments. When learning abstract mathematics, it is extremely helpful to discuss ideas with others, and it can be easier to discern what one does and does not understand when trying to explain things to others. Therefore, collaboration is an indispensable learning tool. However, any work you submit must represent your own effort. Keep the following guidelines in mind when working on homework:

- The solutions that you turn in to me should be written up by you in your own words. It is fine (and encouraged) to discuss ideas with others, but I want each person to think individually about how to put those ideas down on paper.
- If you have worked with others on a particular problem, say so when you write up your solution. If you got a particular idea from someone else, give them the appropriate credit.

In summary, I encourage you to work on homework together, but I do not want you to write up complete proofs as a group—this should be done individually.

Finally, do not present solutions that you have found in other textbooks or on the internet as your own. Aside from committing plagiarism, doing so defeats the purpose of the homework (which is to learn the material through practice). If I determine that you have submitted work that is not your own, I will prosecute plagiarism and academic dishonesty to the fullest possible extent.

COVID-19 Mitigation

As of August 2021, the city of Tyler is in a high-transmission area for COVID-19. Per CDC guidelines and university recommendations, you are expected to wear a face mask covering your nose and mouth during our class meetings. I also ask that you please stay home if you are not feeling well (particularly if you have symptoms like sneezing, coughing, or a higher than normal temperature) at any time during the semester.

Please consult the "University Policies and Information" page on Canvas for more detailed information on current UT Tyler policies regarding COVID-19. You can also visit the UT Tyler website for information on vaccines, testing, and what to do if you have been exposed:

`https://www.uttyler.edu/coronavirus/`

Changes to Syllabus

I reserve the right to make changes to the syllabus during the semester. Any changes to course policies will be announced in class, and an updated version of the syllabus will be posted to Canvas.

Important Dates

- **August 23:** Classes begin.
- **September 3:** Census date. Last day to change schedule or file for grade replacement.
- **September 6:** Labor Day holiday. No class.
- **November 1:** Last day to withdraw.
- **November 22–27:** Thanksgiving break. No classes.
- **December 6:** Study day.
- **December 10:** Final exam.

University Policies

Information on University policies concerning the following topics:

- UT Tyler Honor Code
- Students Rights and Responsibilities
- Campus Carry
- UT Tyler Tobacco-Free Policy
- Grade Replacement/Forgiveness and Census Date
- State-Mandated Course Drop Policy
- Student Accessibility and Resources
- Student Absence due to Religious Observance
- Student Absence for University-Sponsored Events and Activities
- Social Security and FERPA Statement
- Emergency Exits and Evacuation
- Student Standards of Academic Conduct
- UT Tyler Resources for Students

can be found at

`https://www.uttyler.edu/academic-affairs/files/syllabuspolicy.pdf`

Tentative Daily Schedule

This schedule is subject to change as we move through the semester. The topic and textbook section is listed for each class day, and it would be beneficial to read the appropriate section before coming to class. Topics marked with an asterisk (*) are optional and may be skipped or postponed as the schedule permits.

Week	Date	Topics covered	Textbook
1	8/23	Algebra and geometry of complex numbers.	§1.1–1.3
	8/25	Polar form of complex numbers and the complex exponential.	§1.3–1.4
	8/27	De Moivre's formula and powers/roots of complex numbers.	§1.5
2	8/30	Sets in the complex plane.	§1.6
	9/1	The Riemann sphere and stereographic projection.*	§1.7
	9/3	Functions of a complex variable. Limits and continuity for complex functions.	§2.1–2.2
3	9/6	Labor Day – no class.	
	9/8	Complex differentiability and holomorphic/analytic functions.	§2.3
	9/10	Iterated maps: Julia and Mandelbrot sets.*	§2.7
4	9/13	The Cauchy-Riemann equations.	§2.4
	9/15	Harmonic functions.	§2.5
	9/17	Applications of harmonic functions.*	§2.6
5	9/20	Polynomials and rational functions.	§3.1
	9/22	Exponential, trigonometric, and hyperbolic functions.	§3.2
	9/24	The complex logarithm.	§3.3–3.4
6	9/27	Complex powers and inverse trigonometric functions.	§3.5
	9/29	Application to oscillating systems.*	§3.6
	10/1	**Exam 1**	
7	10/4	Prelude to integration: contours in the complex plane.	§4.1
	10/6	Contour integrals.	§4.2
	10/8	Independence of path.	§4.3
8	10/11	Cauchy's Theorem.	§4.4
	10/13	Cauchy's Theorem and the Cauchy Integral Formula.	§4.4–4.5
	10/15	Consequences of the Cauchy Integral Formula.	§4.5
9	10/18	Bounds for analytic functions: Liouville's Theorem and the Maximum Modulus Principle.	§4.6
	10/20	Applications of contour integrals to harmonic functions.*	§4.7
	10/22	Sequences and series of complex numbers and functions.	§5.1
10	10/25	Taylor series and power series.	§5.2–5.3
	10/27	Mathematical theory of convergence for series.	§5.4

	10/29	Laurent series.	§5.5
11	11/1	Zeroes and singularities.	§5.6
	11/3	The point at infinity.*	§5.7
	11/5	Analytic continuation.	§5.8
12	11/8	The Residue Theorem.	§6.1
	11/10	Improper integrals via the Residue Theorem.	6.2–6.3
	11/12	**Exam 2**	
13	11/15	Indented contours.	§6.5
	11/17	Integrals involving multiple-valued functions.	§6.6
	11/19	The Argument Principle and Rouché's theorem.	§6.7
14	11/22	Thanksgiving break – no classes	
15	11/29	Fourier series.*	§6.1
	12/1	The Fourier transform.*	§6.2
	12/3	The Laplace transform.*	§6.3
	12/10	**Final Exam**	